DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010101-Abstract Algebra

Max marks: 50 Time: 1 hour

1. What is the smallest positive integer n such that there are 2 non isomorphic groups of order n ? Name those two groups. .(CO1)
2. How many Abelian groups (up to isomorphism) are there of order 15? .(CO1)
3. Find all sylow 3- subgroups of S_4.(CO2)
4. Let G be a non cyclic group of order 21. How many sylow 3- subgroups does G have? .(CO2)
5. Formulate the field of quotients for $\mathbb{Z}[x]$.(CO3)
6. Find the field of quotients for $\mathbb{Z}_p[x]; p$ is prime.(CO3)
7. Determine all homomorphisms from $\mathbb{R}$ to $\mathbb{R}$.(CO4)
8. .(CO4)
9. Can prime ideals be maximal ideals? Justify. .(CO5)
10. Find prime and maximal ideals for $\mathbb{R}$.(CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010102- Linear Algebra

Max marks: 50 Time: 1 hour

1. Find a basis for the vectorspce consisting of all the polynomials of degree n (CO1)
2. Find a basis for $\mathbb{R}$. Whether $\{0\}$ is a basis? Justify. (CO1)
3. Distinguish between linearly independent and dependent set of vectors. (CO2)
4. Write z linearly independent set and make it dependent by adding one or two more vectors. (CO2)
5. Prove that $T: \mathbb{R}^2 \to \mathbb{R}$ defined by $T(x, y) = x$ is linear. (CO3)
6. Write any two examples for Linear transformation as well as liner functional. (CO3)
7. Find the inverse of $\begin{bmatrix} 1 & 3 & 2 \\ 5 & 7 & 6 \\ 6 & 4 & 2 \end{bmatrix}$ (CO4)
8. Solve $x_1 + 3x_2 = 5$, $2x_1 + 2x_2 = 6$ using Cramer's rule.(CO4)
9. Find a matrix for the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^4$ by $T(e_1) = (3,2,-1,0), T(e_2)$ =(9,4,5,0) $T(e_3) = (1,0,0,-2)$(CO5)
10. Find a matrix for the linear transformation $T: \mathbb{R}^2 \to 2$ by $T(1,1) = (1,2), T(0,-1) = (3,2)$(CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010103- Basic Topology

Max marks: 50 Time: 1 hour

1. Define co finite topology and explain why it is a topology. (CO1)
2. Give three topologies on the real line $\mathbb{R}$ (CO1)
3. Given the collection containing φ and all intervals of the form (q,∞) where q$>\sqrt{2}$ in the real line $\mathbb{R}$. Find whether it is a topology. (CO2)
4. Is the union of two topologies on a given set a topology? Justify your answer. (CO2)
5. Explain the equivalent conditions for continuity of a function and continuity of a function at a point.(CO3)
6. Functions from a discrete space and functions into an indiscrete space are continuous. Explain why it is so?CO3)
7. If a set is connected then find whether its closure is connected? (CO4)
8. Differentiate between connected and disconnected sets and provide examples in the real line (CO4)
9. Prove that if a space is T_1 then it is T_2. Is the converse true? Why? (CO5)
10. Prove that T_0, T_1, T_2, T_3, T_4 form a hierarchy of progressively stronger conditions. (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010104- Real Analysis

Max marks: 40 Time: 1 hour

1. Distinguish between convergence of a sequence & series. (CO1)
2. Write any 3 properties of sequences and series. (CO1)
3. Distinguish between a bounded variation and total variation (CO2)
4. Show that function of a bounded variation is bounded. What about a total variation? (CO2)
5. Formulate the addition property of Riemann- Stieltjes integral (CO3)
6. Evaluate $\int_a^b d \propto (x)\ direct\ from$ the definition of Riemann- Stieltjes integral. (CO3)
7. Evaluate $\lim_{n\to\infty} e - {(1+x)^{1/x}}/{x}$(CO4)
8. Evaluate $\lim_{n\to\infty} (1+x)^{1/x}$

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010105- Graph Theory

Max marks: 50 Time: 1 hour

1. Define: complete grphs, bipartite graphs. Also state first theorem o graph theory. (CO1)
2. Define: digraphs, tournaments, disconnected graphs(CO1)
3. Find vertex connectivity and edge connectivity of $k_{3,2}$ (CO2)
4. Prove that vertex connectivity is less than or equal to edge connectivity in a graph G(CO2)
5. Discuss the concept of “centre” in a tree.(CO3)
6. Explain the concept of minimum length spanning tree. (CO3)
7. Draw K_5 *and determine whether it is planar or not.* (CO4)
8. Draw a graph which is planar but not a plane graph(CO4)
9. Find spectrum of K_n (CO5)
10. Find spectrum of C_n (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010201-Advanced Abstract Algebra

Max marks: 50 Time: 1 hour

1. Prove that $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2}, \sqrt[4]{2})$ is an algebraic extension of $\mathbb{Q}$, but not a finite extension.(CO1)
2. Check whether $\mathbb{R}$ is an extension of $\mathbb{Q}$. What about $\mathbb{C}$ (CO1)
3. If F is a field prove that $F[X]$ is a ED and UFD (CO2)
4. Compare the properties of ED & UD and find whether one implies the other. (CO2)
5. (CO3)
6. (CO3)
7. Find a splitting field o $x^2 + 1$ over $\mathbb{Q}$ (CO4)
8. Find the order of Splitting field of $x^5 + x^4 + 1$over $\mathbb{Z}_2$(CO4)
9. Find the no: of elements in Gal[$\mathbb{Q}(\sqrt[2]{2/\mathbb{Q}}$] (CO5)
10. Find Galois group of $x^2 - 10x + 21$ over $\mathbb{Q}$ (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010202- Advanced Topology

Max marks: 60 Time: $1\frac{1}{4}$ hour

1. Describe why a compact subset of a Hausdorff space is closed. (CO1)
2. Is a regular Lindeloff space normal ? Why?(CO1)
3. State and prove Urysohn characterisation of normality (CO2)
4. State and prove Tietze characterisation of normality. (CO2)
5. Define the cartesian product of finite number of sets and explain how to form the product on an indexed family of sets(CO3)
6. Explain the cartesian product of an indexed family of sets using projection functions (CO3)

7 Determine which of $T_{0,}$, T_1, T_2 are productive ? Explain why? (CO4)

8. Is product of connected space connected? What about the converse?
9. Establish embedding lemma by stating the results used. (CO5)
10. Establish a necessary and sufficient condition for a space to be Tychnoff which uses embedding in a cube (CO5)
11. Explain homotopy of paths and prove that it defines an equivalence relation. (CO6)
12. Explain product of paths by stating the conditions. Also find whether $f * g$ is homotopic to $f' * g'$ if f is homotopic to f' and g is homotopic to g'. (CO6)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010203- Numerical Analysis with Python

Max marks: 45 Time: 1 hour

1. Explain the use of ‘subs’ in a python program (CO1)
2. What are the symbols and expressions used in plotting graph in python? (CO1)
3. Solve $x^2 + 5x + 4 = 0$ using python (CO2)
4. Plot $f(x) = x^3 + 3; x \epsilon \mathbb{R}, |x| \leq 5$ (CO2)
5. Write a program to find the area between two cures. (CO3)
6. Using Lagrange’s interpolation find the curve using following data (CO4)

x	-2	-1	2	5
y=f(x)	-12	-8	3	5

7. Find a real root of $x^3 - x - 1 = 0$ (CO4)
8. Explain Gauss’s elimination method for solving equations (CO5)
9. Explain Choleski decomposition method. (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME01020- Complex Analysis

Max marks: 50 Time: 1 hour

1. Explain Riemann sphere and stereographic projections (CO1)
2. (CO1)
3. Find bilinear transformation which maps $z = 0, -i, -1$ to $w = i, 1, 0$ respectively (CO2)
4. Find bilinear transformation which maps $z = 0, 1, \infty$, to $w = \infty, 1, 0$ respectively (CO2)
5. Evaluate $\int_C z\, e^{z^2} dz$ where C is from 1 to i along the axes. (CO3)
6. Evaluate $\oint_C \frac{e^z}{z}\, dz$; where C consists of $|z| = 2$ counter clockwise (CO3)

   Find the singularities in each of the following(CO4)
7. $\frac{z}{(z+1)(z+2)}$
8. $z - 3\ \frac{1}{(z+2)}$
9. Evaluate $\int_0^\infty \frac{Sin\ x}{x} dx$(CO5)
10. Evaluate $\int_0^\infty Sin\ x^2 dx$ (CO5)

DEVA MATHA COLLEGE, KURAVILANGAD

PG Internal Examination - Semester : II

ME010205: MEASURE THEORY AND INTEGRATION

Total Marks : 50 **Time: 1 hr**

Each question carries 5 marks

1. Construct an open cover for a countable set and compute its Lebesgue outer measure.**(CO1)**
2. Prove that if $m^*(A) = 0$, then $m^*(A \cup B) = m^*(B)$. **(CO1)**
3. Show that the outer measure of an interval is its length. **(CO2)**
4. Using the properties of outer measure, prove that the interval $[0, 1]$ is not countable. **(CO2)**
5. Show that the Dirichlet function is not Riemann integrable over $[0, 1]$ and find its Lebesgue integral. **(CO3)**
6. Does the Bounded Convergence theorem hold for the Riemann integral? Explain. **(CO3)**
7. Let $f: R \to R$be a function that is lebesgue integrable over R. Define the signed measure v as $v(E) = \int_E f\, dm$, on the measurable space (R, L), where L is the σ $-$algebra of Lebesgue measurable sets. Give an Hahn decomposition of R and a Jordan decomposition of v. **(CO4)**
8. State and prove Hahn decomposition theorem and give an example to show that Hahn decomposition is not unique. **(CO4)**
9. Let $A \subseteq X$ and let B be a ν $-$measurable subset of Y . If $A \times B$ is measurable with respect to the product measure $\mu \times \nu$, is A necessarily measurable with respect to μ. **(CO5)**
10. State and prove Tonelli's Theorem. Justify the excision from $X \times Y$ of a set of $\mu \times \nu$ measure zero. **(CO5)**

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010301- Advanced Complex Analysis

Max marks: 50 Time: 1 hour

1. State necessary condition for a function to be Harmonic (CO1)
2. Define a sub harmonic function with an example (CO1)
3. Prove that Laurent development is unique (CO2)
4. If radius of convergence of $\sum a_n z^n$ is R,find radius of convergence for $\sum a_n{}^2 z^n$(CO2)
5. Prove that Riemann Zeta function is analytic in the half plane Re s >1(CO3)
6. Prove that for $\sigma > 1, \zeta(s) = \prod_{p_n, prime} (\frac{1}{1-p_n{}^s})^{-1}$ (CO3)
7. Prove that a family $\mathfrak{I}$ is normal if and only if its closure $\mathfrak{I}$- with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \sqrt{k}(f,g)2^{-k}$ is compact(CO4)
8. If E is a compact set in a region Ω, prove that there exists a constant M depending only on E and Ω such that every harmonic function $u(z)$ in Ω satisfies $u(z_1) \leq Mu(z_2)$ for any two points z_1, z_2 in E(CO4)
9. State and prove Riemann mapping theorem (CO5)
10. Prove that the Weierstrass $\mathcal{P}$ function $\mathcal{P}(z) = \frac{1}{z^2} + \sum_{\omega \neq 0} \left(\frac{1}{(z-\omega)^2} - \frac{1}{\omega^2}\right)$ (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010303- Multivariate Calculus & Integral Transforms

Max marks: 50 Time: 1 hour

1. State and prove Weirstrass approximation theorem. (CO1)
2. Describe the general form of Fourier series (CO1)
3. Define total derivative and Jacobian matrix (CO2)
4. Show that if **g** is differentiable at **a** and **f** is differentiable at **b=g(a)** then the composition function **h=f**o**g**, is differentiable at **a** (CO2)
5. Assume that one of the partial derivatives $D_1 f, D_2 f, \dots D_n f$ exist at c and the remaining n-1 partial derivatives exist in some n-ball B(c). Prove that f is differentiable at c(CO3)
6. Show that Cauchy Riemann equations along with differentiability of u and v imply existence of $f'(c)$ (CO3)
7. Define stationary point and saddle point(CO4)
8. Find and classify the extremum values (if any) of $f(x,y) = x^2 + y^2 + x + y + xy$(CO4)
9. Define a differential *k*-form and its elementary properties(CO5)
10. If $r(t) = (a\cos t, b\sin t), 0 \le t \le 2\pi$. Find $\int_r x\,dy$ and $\int_r y\,dx$ (CO5)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010304- Functional Analysis

Max marks: 40 Time: 1 hour

1. Give example for Normed spaces that are not Banach spaces with justification (CO1)
2. Prove that $[a,b] \subseteq \mathbb{R}$ is compact. (CO1)
3. How does linear functional differ from linear operators? (CO2)
4. Give an example for a linear operator and linear functional on $\mathbb{C}$ over $\mathbb{R}$ (CO2)
5. Explain the concept of Innerproduct spaces with exaples (CO3)
6. Define Hilbert spaces. What do you mean by 'direct sum' in Hilbert spaces (CO3)
7. Distinguish between orthonormal sets & sequences. (CO4)
8. State any 3 properties of orthonormal sequences (CO4)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010305- Optimization Techiques

Max marks: 50 Time: 1 hour

1. Solve using simplex method (CO1)
   Minimize $-5x_1 - 3x_2$ such that $x_1 + x_2 \leq 2$, $5x_1 + 2x_2 \leq 10$, $3x_1 + 8x_2 \leq 12$, $x_1, x_2 \geq 0$
2. Prove that dual of the dual is primal (CO1)
3. Solve using ranch &bound method:(CO2)
   Maximize $x_1 + 2x_2$
   Such that $5x_1 + 7x_2 \leq 21, -x_1 + 3x_2 \leq 8, x_1, x_2 \geq 0$
4. Solve using cutting plane method: (CO2)
   Maximize $x_1 + 2x_2$
   Such that $5x_1 + 7x_2 \leq 21, -x_1 + 3x_2 \leq 8, x_1, x_2 \geq 0$
5. Explain the concept of maximum flow in a network(CO3)
6. Discuss goal programming in detail.(CO3)
7. Solve using gradient projection method (CO4)

$$Min\ f(x) = 4x_1 + {x_2}^2 - 6$$

$$\text{Such that } 26 - {x_1}^2 - {x_2}^2 = 0$$

8. Explain Lagrange’s multiplier’s method & Gradient projection method (CO4)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010302- Partial Differential Equations

Max marks: 50 Time: 1 hour

1. Define Pffaffian differential forms. What do you mean by an integrating factor of a differential equation (CO1)
2. Explain Pffaffian differential equations in 2 variables and discuss when it becomes exact.(CO1)
3. Distinguish between linear and non-linear differential equations (CO2)
4. Solve $\sin x\left(\frac{dy}{dx}\right)+3y=\cos x$ (CO2)

   Find the complete integral using Charpit's method for following (CO3)
5. $(p^2+q^2)y=qz$
6. $p^2x+q^2y=z$

   Find the complete integral using Jacobi's method for following (CO4)
7. $z^2=pqxy$
8. $z(y+zp)=q(xp+yq)$
9. Solve the wave equation: $r=at^2$
10. Solve $z(qs-pt)=pq^2$

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

DISSERTATATION

Max. Marks: 20

1. What is the basic concept /reason which caused you to choose the topic?(CO1) (5 Marks)
2. Explain the relevance of your project in present research scenario?(CO1) (5 Marks)

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3. Present your idea and concepts that caused you to choose this topic?(CO2) (5 Marks)
4. Explain the applications o your project.(CO2) (5 Marks)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME010401- Spectral Theory

Max marks: 50 Time: 1 hour

1. Discuss strong, weak and uniform convergence of sequence of operators on a Normed space X (CO1)
2. State bounded inverse theorem. Write any o its consequences (CO1)
3. Find a fixed point of the function $f(z) = z^2$ (CO2)
4. Find the eigen values for the following (CO2)
$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$
5. Prove that similar matrices have same eigen values (CO3)
6. Prove that resolvant set of a bounded linear operator of a complex Banach space is open (CO3)
7. Prove that identity operator I is not compact in infinite dimensional normed spaces. (CO4)
8. Let $T: l^2 \to l^2$ defined by $T(\varepsilon_1, \varepsilon_2, \dots) = (\varepsilon_1, \frac{\varepsilon_2}{2}, \dots \frac{\varepsilon_n}{n}, 0, 0 \dots)$. Prove that T is compact(CO4)
9. Prove that eigen values are always real for a bounded self adjoint linear operator. (CO5)
10. Does all bounded linear operators on a Hilbert spaces are projection? Justify (CO5)

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Outcome Measurement Test

ME010402- Analytic Number Theory

Max marks: 50 Time: 1 hour

1. Distinguish between Mobius function and Euler totient function. (CO1)
2. Distinguish between multiplicative and completely multiplicative functions (CO1)
3. Find average order of $\sigma_\alpha(n)$ (CO2)
4. Find average order of d(n) (CO2)
5. State Shapiro-Tauberian theorem (CO3)
6. Write any equivalent statements for explaining limiting properties for primes. (CO3)
7. Solve $49x \equiv 28 (mod 119)$ (CO4)
8. Find $\phi(10)$ and reduced residue modulo 10 (CO4)
9. State qudratic law of reciprocity(CO5)
10. Explain primitive roots of n(CO5)

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Outcome Measurement Test

ME800401-Differential Geometry

Max marks: 50 Time: 1 hour

1. Define Tangent space and show that tangent vectors are orthogonal to the gradient .(CO1)
2. Define Level set and graph of function. Give examples? .(CO1)
3. Explain the parallel transport and properties of parallelism. CO2)
4. Discuss the spherical image of an n-surface with Orientation **N** is the reflection through the origin of the spherical image of the same n-surface with Orientation -**N** .(CO2)
5. Compute the Weingarten map for the circular cylinder $x_2^2 + x_3^2 = a^2$ in R^3 $(a \neq 0)$.(CO3)
6. Find the curvature k of the oriented plane curve for $x_1^2 - x_2^2 = 1, x_1 > 0$.(CO3)
7. Are local parameterization of plane curve unique up to reparameterization ? Justify your answer. (CO4)
8. Let S be a compact oriented n-surface in R^{n+1}. Examine whether there exist a point p such that the second fundamental form of p is definite.(CO4)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

ME800403- Combinatorics

Max marks: 40 Time: 1 hour

1. Explain the concepts of permutations and combinations (CO1)
2. Find the no: of ways to arrange n objects in a row and find the no:of ways to choose 3 people from n people. (CO1)
3. Prove that in a group of 7, there must be atleast 4 of the same sex. (CO2)
4. Prove that in a gathering of 6 people, some 3 of them are either mutual acquaintances or totally strangers. (CO2)
5. State principle of inclusion and exclusion (CO3)
6. Using PIE , find the no: of integers from {1,2,…500} which are divisible by 2,3 or 5. (CO3)
7. Find generating function for the sequence (1,2,3,…) (CO4)
8. Find the coefficient of $x^k\,; k \geq 18$ in the expansion of $(x^3 + x^4 + x^5 + \cdots)^6$ (CO4)

DEVAMATHA COLLEGE KURVILANGAD

Outcome Measurement Test

Viva-Voce

Max marks: 30

1. Explain the concept of topological properties in a layman concept. (CO1)
2. What do you mean by analyticity of a function? And explain singularities with examples. (CO1)
3. Explain norm, normed spaces, inner product and Hilbert spaces. (CO1)
4. What do you mean by a plane graph? (CO1)
5. Explain the concept of convergence of a sequence in real line, complex plane, normed spaces and generally on a topological spaces.(CO2)
6. How does the concept of homomorphism varies in groups, rings and vector spaces?(CO2)